The Sonnenschein-Mantel-Debreu Results after Thirty Years

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The Impact of the Sonnenschein-Mantel-Debreu Results

The properties of aggregate excess demand or market demand play a key role in the existence, uniqueness, stability, and comparative statics of competitive general equilibrium. They are also important for the theory of imperfectly competitive general equilibrium and the microfoundations of macroeconomics. Because observations of demand are likely to occur at the aggregate level, whereas economic theory has been largely concerned with individual behavior, the properties of aggregate excess or market demand are central to econometric identification. The importance of these projects for the development of general equilibrium theory can be seen in Kenneth Arrow and Frank Hahn's *General Competitive Analysis* (1971).

Soon after their book was published, though, Hugo Sonnenschein (1973) posed the following problem. Consider a function that maps prices into quantities. What conditions must this function satisfy if it is to be the aggregate excess demand function of a well-behaved exchange economy? The existence theory highlighted the importance of the function's satisfying continuity, homogeneity of degree zero, and Walras's law. Were these all of the restrictions on aggregate excess demand implied by the usual

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assumptions on preferences and endowments? Sonnenschein conjectured that this was true when he asked, "Do Walras' identity and continuity characterize the class of community excess demand functions?" He showed the answer to be "yes" for the case of two goods. Rolf Mantel (1974) and Gerard Debreu (1974) proved general versions of these results on aggregate excess demand. Debreu's work used less-stringent assumptions than Mantel's and proved the following theorem: for a function that is continuous, homogeneous of degree zero, and in accord with Walras's law, there is an economy with at least as many agents as goods such that, for prices bounded away from zero, the function is the aggregate demand function for this economy. This is the Sonnenschein-Mantel-Debreu (SMD) theorem.

There followed elaborations and clarifications. Mantel (1976) showed that the theorem was valid even if agents had homothetic preferences (and arbitrary endowments); thus a common assumption made in an attempt to gain tractability (see Chipman 1974) did not help in this context. Even the intuition that collinear endowments would help turned out to be false. Alan Kirman and K. J. Koch (1986) showed that the assumption that the income distribution is fixed in this way does not restrict excess demand. For any smooth function that is homogeneous of degree zero, which satisfies Walras's law, and is defined on the strictly positive orthant, there is an economy of agents with smooth, monotone, and strictly convex preferences whose endowments are collinear such that the economy's aggregate demand coincides with the function on the strictly positive orthant. The SMD results also seemed to imply that the set of equilibrium prices did not have restrictions. In fact, Andreu Mas-Colell (1977) showed that given any nonempty compact set of prices (that are strictly positive), it is always possible to construct an economy of consumers with continuous, monotone, and strictly convex preferences such that the economy's equilibrium prices coincide with the given set. Thus observations on market prices alone do not restrict in any meaningful way the sort of economy that could have generated them. Erwin Diewert (1977), in a differentiable setting, did find some restrictions on the derivatives of aggregate excess demand, but John Geanakoplos and Herakles Polemarchakis (1980) showed that these are the only restrictions. For some time the problem of market demand functions (where individuals have given incomes rather than given endowments) was not completely solved. However, Diewert (1977) and Mantel (1977) made progress on this problem, and Jordi Andreu (1982) showed that SMD-type results apply to finite subsets of prices for market

demand. Pierre-André Chiappori and Ivar Ekeland (1999), using smoothness assumptions, showed that SMD results extend to the whole of the market demand function. The corpus of SMD theory, therefore, is fairly complete. Along with the Arrow-Debreu existence theorem and some results on regular economies, SMD theory fills in many of the gaps we might have in our understanding of general equilibrium theory (Chiappori et al. 2004).[1]

It is also a deeply negative result. SMD theory means that assumptions guaranteeing good behavior at the microeconomic level do not carry over to the aggregate level or to qualitative features of the equilibrium. It has been difficult to make progress on the elaborations of general equilibrium theory that were put forth in Arrow and Hahn 1971. There are problems with establishing general results on uniqueness (Ingrao and Israel 1990, chap. 11; Kehoe 1985, 1991; Mas-Colell 1991), stability (Sonnenschein 1973; Ingrao and Israel 1990, chap. 12; Rizvi 1990, 94–144), comparative statics (Kehoe 1985; Nachbar 2002, 2004), econometric identification (Stoker 1984a, 1984b), microfoundations of macroeconomics (Kirman 1992; Rizvi 1994b), and the foundations of imperfectly competitive general equilibrium (Roberts and Sonnenschein 1977; Grodal 1996). Subfields of economics that relied on well-behaved aggregate excess demand for much of their theoretical development, such as international economics, were also left in the lurch (Kemp and Shimomura 2002).

In part because of a conviction that progress could not be made in general equilibrium theory, there was a substantial redirection in economic theory. As the results in SMD theory became well known, for example through Wayne Shafer and Hugo Sonnenschein's survey (1982), economists began to question the centrality of general equilibrium theory and put forward alternatives to it. Thus in the ten years following the Shafer-Sonnenschein survey, we find a number of new directions in economic theory. It was around this time that rational-choice game theory methods came to be adopted throughout the profession, and they represented a thoroughgoing change in the mode of economic theory. Even so, following a growing realization of formal difficulties with rational-choice game theory as well as experimental evidence that did not agree with some of its predicted outcomes, a group of practitioners turned to evolutionary game theory. Indeed, the rise of experimental economics itself rep-

1. Wayne Shafer and Hugo Sonnenschein (1982) provide a survey of SMD theory. Andreu Mas-Colell, Michael Whinston, and Jerry Green (1995, 598–606) give a more recent treatment.

resents an important development in the growth of alternative approaches in the wake of general equilibrium theory's difficulties. Alongside these developments, other approaches that emphasized less than fully rational behavior, such as behavioral economics, also gained prominence (Sent 2004). It was now possible to see, with more frequency, models whose agents were hardwired to act in certain ways or who had "zero intelligence" (Gode and Sunder 1993).

Others placed the source of the problem at one of the characteristic modes of procedure in much of economic theory, namely, the "idea that we should start at the level of the isolated individual." Instead, the goal should be "to theorise in terms of groups who have collectively coherent behaviour" (Kirman 1989, 138) so that interaction among individuals leading to regularities at the aggregate level does not operate solely through markets. This suggests an analysis of complex systems (Kirman 2004; see also Saari 1995). Or theorists might make assumptions about the distribution of agents' characteristics, in which case "assumptions about the organisation of society" (Kirman 1989, 138) might explain other regularities at the macro level. This is the approach taken in what might be called the European approach to market demand associated with Werner Hildenbrand, Jean-Michel Grandmont, and others. A characteristic problem that these authors pursue is whether a declining distribution of income (richer individuals are less numerous), or other dispersions of agents' characteristics such as preferences, leads to a justification for the law of demand in the aggregate (Hildenbrand 1994; Grandmont 1992; Quah 2000).

We see, then, that the difficulties that led to a realization that there were pervasive problems in the general equilibrium project also led to a redirection of economic theory. What emerges is less and less recognizable as being in the tradition of the microeconomics that dominated in the postwar era (Rizvi 2003). We might even say that this has led to pluralism in economics after a long period in which valid approaches were seen to be those that sought their basis in general equilibrium theory.[2]

Given how sweeping the changes wrought by SMD theory seem to be, it is understandable that some very broad statements about the character

2. I have considered the developments mentioned in this paragraph and the previous two in a series of writings (Rizvi 1994a, 1997, 1999, 2003, 2005a, 2005b). A number of other authors concerned with the history and development of recent economics have commented perceptively on these transformations in the theory. A partial list would include Mirowski 1993, 312–13, 333–35; Mirowski and Hands 1998; Sent 2005; Giocoli 2003, 204, 377, 408; and Davis 2003, 37–38.

of general equilibrium theory were made. Fifteen years after *General Competitive Analysis*, Arrow (1986) stated that the hypothesis of rationality had few implications at the aggregate level. Kirman (1989) held that general equilibrium theory could not generate falsifiable propositions, given that almost any set of data seemed consistent with the theory. These views are widely shared. Bliss (1993, 227) wrote that the "near emptiness of general equilibrium theory is a theorem of the theory." Mas-Colell, Michael Whinston, and Jerry Green (1995) titled a section of their graduate microeconomics textbook "Anything Goes: The Sonnenschein-Mantel-Debreu Theorem." There was a realization of a similar gap in the foundations of empirical economics. General equilibrium theory "poses some arduous challenges" as a "paradigm for organizing and synthesizing economic data" so that "a widely accepted empirical counterpart to general equilibrium theory remains to be developed" (Hansen and Heckman 1996).[3] This seems to be the now-accepted view thirty years after the advent of SMD theory.

However this may be, there is a recent challenge to this now-conventional wisdom. The key work that has led to a reconsideration of the importance of SMD theory is Donald Brown and Rosa Matzkin's (1996) paper about testable restrictions on the equilibrium manifold. They and their supporters claim that general equilibrium theory does indeed have scientific status and that it can make progress on some of the issues where the aggregate demand approach proved ineffectual. Chiappori et al. (2004, 106) hold that Arrow's view "is overly pessimistic, and that general equilibrium theory can actually generate strong testable predictions." In the remainder of this article, I lay out the Brown-Matzkin results and some variants. I then consider how we should see their work in assessing the impact of SMD theory after thirty years.

Testable Restrictions on Equilibrium Manifold

Brown and Matzkin's work was motivated by the concern that general equilibrium theory lacks falsifiable implications or, as Paul Samuelson

3. These conclusions also elicited negative reactions from those who had counted on general equilibrium theory to provide a progressive structure for theory and policy. Werner Hildenbrand (1994, preface) says that the SMD theory left him "deeply consternated." Herakles Polemarchakis (2004, 140) reports, "Rumour has it that James Tobin, who held strongly that economics can and should alleviate need and improve general welfare, considered the result of Sonnenschein-Debreu-Mantel as a result that should not have been proved."

put it, "meaningful theorems" (cited in Brown and Matzkin 1996, 1249).[4] For Samuelson (1947, 4) such theorems are hypotheses "about empirical data that could conceivably be refuted."[5] In considering their project, we might recall the charge by Kirman (1989, 126, 127) that general equilibrium theory is "empty in the sense that one cannot expect it to house the elements of a scientific theory, one capable of producing empirically falsifiable propositions" or, similarly, that "it seems we have arrived at the point where the current model is shown to be *intrinsically* incapable of generating verifiable propositions." Or we might recall Bliss's statement that the "emptiness of general equilibrium theory is a theorem of the theory."

It is important to understand what authors like Kirman and Bliss did *not* mean. Aspects of general equilibrium theory do have verifiable propositions, but these are at the level of individuals (and so refer to a situation prior to a general equilibrium). Thus there are the Slutsky restrictions, or the weak axiom of revealed preference (WARP), at the individual level. Yet these do not carry over to the aggregate realm. Thus Kirman and others could not have meant individual-level propositions. Similarly, Arrow spoke of the implications of rationality in the aggregate. A statement of his has come under scrutiny because of the Brown-Matzkin work. Arrow (1986, S388) stated, "In the aggregate, the hypothesis of rational behavior has in general no implications," concluding that "if agents are different in unspecifiable ways, then . . . very little, if any, inferences can be made" (S389). This will be important later on. Finally, a last preliminary point. The SMD results are theorems and so cannot be wrong. This means that results that seem to disagree with them must be based on different constructs or premises. What is at stake is what this altered framework implies. It is important, then, to assess how the SMD theory and the Brown-Matzkin results can coexist.

What, then, are the Brown-Matzkin claims?[6] Brown and Matzkin, rather than look at aggregate excess demand, consider the equilibrium manifold

4. An earlier title for their paper was "Walrasian Comparative Statics." Thus their concern extends to the qualitative features of general equilibrium. This aspect of their work is considered below.

5. I pass over the issue of whether refutability or falsification makes sense as a test of the validity of general equilibrium theory. Polemarchakis (2004) argues that it does. John Chipman and Jean-Sébastien Lenfant discuss prior attempts to test demand theory and the issue of aggregation (2002, 568 n. 30, and accompanying discussion). On these issues generally, see Hands 1993.

6. There are extensions of the Brown and Matzkin paper, though not all of them yield positive results. Felix Kubler (2003) looks for testable restrictions in an expected utility

(Balasko 1975): given preferences, the equilibrium manifold is the set of endowments and prices for which the market excess demand is zero. As Andres Carvajal, Indrajit Ray, and Susan Snyder (2004, 3) point out, the equilibrium manifold may be a natural way to think of testable restrictions arising out of general equilibrium theory, since the exogenous variables, such as endowments, can be varied to see their effects on the endogenous variables, the prices.

It might be easiest to see Brown and Matzkin's argument as a series of steps.

First, they begin with results from revealed preference theory. Sidney Afriat asked when a finite number of observations on prices and quantities could be consistent with utility maximization. He developed a condition called cyclical consistency as well as a set of inequalities that are now called the Afriat inequalities to answer this question. The Afriat inequalities contain unobservable utility levels (U^i), marginal utilities of income (μ^i), and consumption bundles.[7] Using the theorem of the alternative, Afriat showed the inequalities to be equivalent to a cyclical consistency condition that contains only observable prices and quantities. This can be stated more formally. Given N observations on prices and quantities (p^i, x^i), $i = 1, \ldots N$, a utility function $U(x)$ *rationalizes the data* if for all x, $p^i \cdot x^i \geq p^i \cdot x$ implies $U(x^i) \geq U(x)$. Next, the following theorem of Afriat, as stated by Hal Varian (1982, 946), shows that a number of statements are equivalent.

Theorem (Afriat 1967):

Given a set of data, (p^i, x^i), $i = 1, \ldots N$ *the following conditions are equivalent*:

- *There exists a non-satiated utility function that rationalizes the data.*
- *The data satisfy cyclical consistency, i.e., for all* $\{r, s, t, \ldots, q\}$ *if* $p^r \cdot x^r \geq p^r \cdot x^s$, $p^s \cdot x^s \geq p^s \cdot x^t, \ldots,$ *and* $p^q \cdot x^q \geq p^q \cdot x^r$, *then* $p^r \cdot x^r = p^r \cdot x^s$, $p^s \cdot x^s = p^s \cdot x^t, \ldots,$ *and* $p^q \cdot x^q = p^q \cdot x^r$.

framework, Susan Snyder (1999) considers public goods, and Andres Carvajal (2003), externalities. Carvajal, Indrajit Ray, and Snyder (2004) and Pierre-André Chiappori, Ivar Ekeland, Kubler, and Polemarchakis (2004) summarize and develop this literature.

7. Brown and Matzkin consider consumption bundles to be unobservable but endowment bundles to be observable. They realize that endowment bundles are difficult to observe and so develop the theory also in terms of income rather than endowments, as is explained below.

- *There exist numbers* $U^i > 0$, $\mu^i > 0$, $i = 1, \ldots N$, *that satisfy the Afriat inequalities*: $U^i \leq U^j + \mu^j p^j \cdot (x^i - x^j)$ *for* $i, j = 1, \ldots N$.
- *There exists a concave, continuous, non-satiated utility function that rationalizes the data.*

Afriat's theorem thus relates utility maximization and revealed preference theory.[8]

Second, given Afriat's theorem, what conditions must hold in equilibrium? Suppose there are T agents with the well-behaved preferences specified in Afriat's theorem. Suppose also that we have N profiles of agents' utility levels (U^i), marginal utilities of income (μ^i), and consumption bundles (x^r) (they can be thought of as the unobserved data that correspond to observations on prices and endowments). Brown and Matzkin (1996, 1253) note that, given their assumptions, the following conditions will hold in equilibrium (the agent-specific subscripts are dropped):

- $U^i \leq U^j + \mu^j p^j \cdot (x^i - x^j)$ for $i, j = 1, \ldots N$ (the Afriat inequalities)
- $\mu^i > 0$, $x^i \geq 0$, $i = 1, \ldots N$ (positive marginal utility of income, nonnegative consumption)
- $p^i \cdot x^i = p^i \cdot w^i$ *for* $i = 1, \ldots N$ (satisfied budget constraints)
- $\sum x^i = \sum w^i$, summed over all T agents for $i = 1, \ldots N$ (markets clear)

They call these the *equilibrium inequalities*. The observable variables are the endowments and prices. These restrictions together form a family of polynomial inequalities in the unobserved variables (utilities, marginal utilities, and consumption bundles).

Third, Brown and Matzkin invoke a result from model theory called the Tarski-Seidenberg theorem that proves that any finite system of polynomial inequalities can be reduced to an equivalent finite family of polynomial inequalities in the coefficients of the system, with an algorithm involving a finite number of steps (Van Den Dries 1988). Equivalence means that the original system of inequalities has a solution if and only if values of the coefficients satisfy the derived system of inequalities. (Indeed, the equivalence of the Afriat inequalities and cyclical consistency [which contains only the coefficients of the system, namely, the observable prices and endowments] is an instance of the Tarski-Seidenberg theorem.) In the case of the equilibrium inequalities, the coefficients of

8. Hal Varian (1982) showed that cyclical consistency is equivalent to having the data satisfy the generalized axiom of revealed preference (GARP), thus adding to this list of equivalent conditions in the theorem.

the system are the observable prices and endowments, and the family of inequalities is composed of the Afriat inequalities, budget constraints, and market-clearing conditions.

Fourth, there are now three possibilities.

1. The Tarski-Seidenberg algorithm does not terminate because the equilibrium inequalities do not have a solution. In this case equilibrium does not exist.
2. The algorithm terminates, but the inequalities are satisfied by every observation on prices and endowments. In this case, the equilibrium model cannot be refuted by the data, since all the data are consistent with it.
3. The algorithm terminates, and the polynomial inequalities define a strict subset of the price-endowment space (the equilibrium manifold). In this case, the equilibrium model is refutable, since price-endowment observations may be inconsistent with equilibrium.

Fifth, Brown and Matzkin now conclude their argument. Point 1 is not true because of the Arrow-Debreu existence theorem. This leaves points 2 and 3. They rule out point 2 by means of a counterexample. They show an example of two observations of prices and endowments that violate cyclical consistency. Figure 1 shows two different observations of endowments and prices. The rectangles are Edgeworth boxes with sides equal to the sum of endowments of two goods for two agents. Agent 1's origin is at *A* for the first observation on endowments (*ABCD*), and prices are shown by the line going through *a* and *b*. The line segment *ab* shows the allocations that are potentially equilibrium allocations. Agent 2's origin is at *C* for the first observation, and at *F* for the second (*AEFG*); agent 1's origin remains at *A*; and the line through *c* and *d* shows prices for the second observation. Cyclical consistency is violated by these two sets of data, since every allocation in the second Edgeworth box is available at the first set of prices, and every allocation in the first box is available at the second set of prices. Cyclical consistency is equivalent to the Afriat inequalities (by Afriat's theorem), which is necessary for equilibrium. This means that point 2 cannot be true. Hence point 3 follows, and the model is refutable.[9]

9. Chiappori et al. report a complementary result based on the equilibrium manifold. In a differentiable framework, they prove that, if income effects remain positive, observing the equilibrium manifold "generically identifies the underlying economy, in the sense that individual

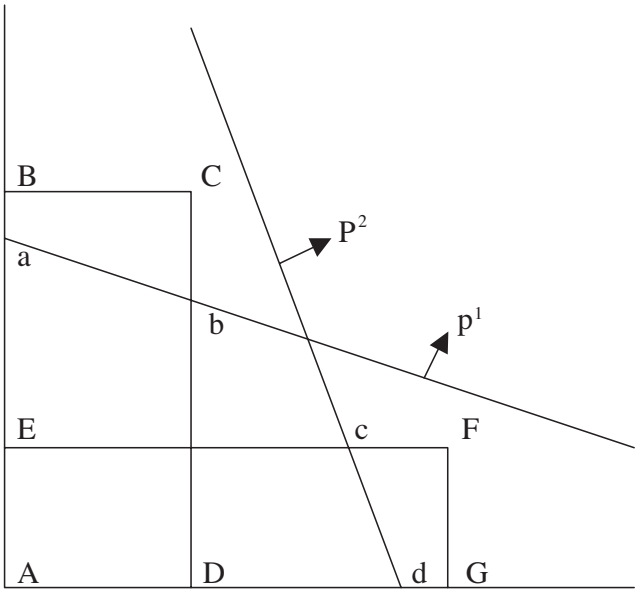

Figure 1 A violation of cyclical consistency

Implementation

Brown and Matzkin advance the possibility that their work can be implemented empirically. As I have shown, the restrictions they find are in the form of a system of nonlinear polynomial inequalities in prices and endowments. Since endowment vectors may be hard to observe, Brown and Matzkin restate their results in terms of income rather than endowments. This improves the possibility that their work can serve to test general equilibrium theory, since the observable variables are now prices

preferences can be recovered without ambiguity" (Chiappori et al. 2004, 107). This, they say, provides a conclusion opposite to SMD theory, since the microeconomic structure of preferences is recoverable rather than being lost by observations on the equilibrium manifold. However, in the SMD results, it is the aggregate structure that is unavailable. Moreover, the approach of Chiappori et al. requires considerable information (in particular, microeconomic data on endowments or income, an issue discussed below), and their work differs from Brown and Matzkin, who require only a finite set of data; this, as Carvajal (2002, 4) says, is "more convenient from an empirical perspective."

of all goods and incomes of all consumers. In addition, they refer to their work on random preferences, work that raises the possibility that the restrictions could be identified in the form of simultaneous equations and then tested. Results analogous to theirs have been developed for random preferences (Carvajal 2003, cited in Carvajal, Ray, and Snyder 2004). Further, Brown and Matzkin derive exact restrictions for a number of special cases: for two agents and two observations, homothetic preferences, and a Robinson Crusoe production economy.

Nevertheless, there remain problems of implementation. First, regarding their special cases, they say that in the "stylized economies in our examples one should think of each 'trader' as an agent type, consisting of numerous small consumers each having the same tastes and incomes" (Brown and Matzkin 1996, 1257). This interpretation raises the same questions of implementation that led researchers to look to aggregate excess demand or to market demand in the first place, so as not to have to assume that individuals simply represented aggregates by the fiat of the theorist (Kirman 1989, 1992; Rizvi 1990, 143 n. 14). Second, to apply the method to large data sets, researchers would need an efficient way to solve large systems of nonlinear polynomial inequalities. Brown and Matzkin (1996, 1258) suggest using particular functional forms for utility to reduce the complexity of the problem. Third, the method they propose requires estimating the impact of changes of individual endowments or incomes on aggregate prices. The larger the economy, the smaller such effects are likely to be and the harder it will be to produce empirical work based on the observed variation (Chiappori et al. 2004, 117–18). Chiappori et al. echo Brown and Matzkin's idea of interpreting agents as types but also argue that "general equilibrium does not apply only to 'large' economies" and suggest applications to small groups such as "committees, clubs, villages and other local organizations" (118), citing Townsend 1994 as an example. It is difficult to agree wholeheartedly. Large economies are precisely those in which the price-taking assumption of competitive equilibrium theory makes most sense and where convexifying effects make agent-level nonconvexities and discontinuities irrelevant to existence theory.

Interpretation

Whether or not the Brown-Matzkin results can be implemented successfully, researchers have established that, given certain kinds of data, general

equilibrium theory can be falsified. This stands in contrast to the usual interpretation of the SMD results, according to which it seems that the aggregate manifestations of general equilibrium theory are vacuous. The contrasting implications for the interpretation of general equilibrium theory require some discussion. It is clear that the two sets of results refer to different constructs. The SMD results refer to aggregate demand (excess or market) and the Brown-Matzkin theory to the equilibrium manifold. Thus we can see why these results coexist. This still leaves open matters of interpretation.

The first point to consider is that the two theories do not refer to the same type of data. In the case of SMD theory, the data were all at the aggregate level. The theory refers to *aggregate* excess demand and prices. This is why Arrow (1986, S388) said, "In the aggregate, the hypothesis of rational behavior has in general no implications." His statement is still correct. The Brown-Matzkin hypothesis requires at least two observations of individual-level data (endowment vectors or incomes) to proceed, as does any other approach that works with the equilibrium manifold.[10] This accounts for one difference between the two approaches. The question then arises whether individual-level data are necessary for the refutability of general equilibrium theory. The answer seems to be "yes."

Chiappori and Ekeland (1999) prove this point as follows (see also Chiappori et al. 2004, 115–17). Suppose only an aggregate endowment is observed, and it is assumed that there is some rule that maps the aggregate endowment into individual endowments, but those are not observed.[11] However, the prices that result are also observed. As the endowment distribution

10. Mas-Colell, Whinston, and Green, in their discussion of the SMD results, point out that the theory does not restrict endowments. By contrast, Brown and Matzkin (1996, 1252) assume that consumption is from the nonnegative orthant and endowments are strictly positive. The SMD procedure of constructing a well-behaved economy that can generate a given aggregate excess demand is not always possible if endowments are restricted (Mas-Colell, Whinston, and Green 1995, 604, fig. 17.E.3). Mas-Colell, Whinston, and Green differentiate the SMD approach from that of Brown and Matzkin, referring to an earlier version of the paper I have discussed. In a general approach to comparative statics, "any first-order effect is possible . . . [yet] it is also the case that if there are prior restrictions on initial endowments and if consumption must be nonnegative, then there are again comparative statics restrictions of a global character" (Mas-Colell, Whinston, and Green 1995, 617; see Brown and Matzkin 1993 for a recent investigation of this point). Here it is important to note Nachbar's (2000, 2074) observation that the Brown-Matzkin restrictions, in the context of comparative statics analysis, "have not yielded an easy interpretation."

11. If we observe only aggregates, such as aggregate endowments, there nevertheless has to be a way to allocate the aggregate endowment to agents. This is what the rule does.

rule changes, there are fluctuations in prices. Then the equilibrium manifold is a function of aggregate endowments and a distribution rule. Is there a restriction on the form of this relationship? Chiappori and Ekeland show that when the number of agents is at least as large as the number of goods (a standard assumption from SMD theory), any smooth-enough manifold can be the result of utility maximization for a distribution rule satisfying certain conditions.[12] Thus individual-level data are necessary for testable restrictions to follow from the equilibrium manifold approach (Chiappori et al. 2004, 115–17). So we see that the issue of refutability depends on what kinds of data are observed. If we stick to nonindividual or aggregated data (prices, endowments, consumption), then general equilibrium theory cannot be refuted. But if we have enough individual-level data, it can be, at least in principle. There is no inconsistency between the SMD theory and the Brown-Matzkin results on this count. As Brown and Matzkin (2000, 1529) state, "No statement concerning refutable implications is meaningful without specifying what information is observable and what is unobservable."

Are the Brown-Matzkin results then trivial? Do they involve putting the individual-level data into a hat only to pull out refutability later? I do not think so. Individual-level data on incomes or endowments alone are not obviously connected to prices. Nor do individual choices have to be observed in the Brown-Matzkin approach; only individual constraints do. Thus the Brown-Matzkin theory does represent a crack in the edifice of interpretation built on the SMD theory.

Yet it is important not to conclude too much, either. Even if we grant that implementation problems are no impediment, and allow individual-level data, refutability is only one aspect of a desirable theory (and see the reservation expressed in footnote 5). The SMD results challenged many others. What, for example, is the impact of the Brown-Matzkin results on uniqueness, stability, and comparative statics?

Uniqueness, Stability, and Comparative Statics

Do the new approaches allow us to test uniqueness, stability, or comparative statics? The answer to this question is "no." Suppose that we had data on incomes and prices, as in the original Brown and Matzkin (1996)

12. In addition to smoothness, they also assume an analyticity condition.

contribution. Then for us to conclude that the data, for example, showed local instability, which is certainly theoretically possible, an economy that rationalized the data would have to be one in which local instability was possible. The same would be true for other qualitative features of equilibrium. Yet it turns out that rationalizing the data puts no significant restrictions on qualitative features of equilibrium (uniqueness, stability, and comparative statics). Four years after the Brown-Matzkin contribution, Donald Brown and Chris Shannon (2000) demonstrated the following result for an exchange economy. Begin with a series of observations on prices, incomes, and aggregate consumption. Then these data can be rationalized by an economy in which each consumer has a smooth, strictly quasi-concave, and monotone utility function if and only if they can be rationalized by an economy with consumers having those preferences (smooth, strictly quasi-concave, monotone) and in which each observed equilibrium is locally unique and locally stable under tâtonnement, and the equilibrium correspondence is locally monotone. Thus the data can be rationalized at all if and only if they can be rationalized for an economy in which equilibrium has well-behaved qualitative features. Thus the data could never reject the hypothesis that the economy had problematic qualitative features. What this means is that "local uniqueness, local stability, and local monotone comparative statics are not refutable given a finite set of observations on prices, income levels, and aggregate consumption," and this is true even if individual demands are observed (Brown and Shannon 2000, 1539).

Conclusion

We are now in a position to sum up the effect of Brown and Matzkin's work on the conclusions drawn from the SMD results. Brown and Matzkin show that it is in principle possible for general equilibrium theory to generate refutable restrictions. These are in the form of polynomial inequalities on the equilibrium manifold where the observables are endowment vectors (or incomes) and prices. It may be difficult to perform tests along these lines as the dimensions of the problem increase. This is all the more likely to happen in a setting where the number of agents is large enough to justify the price-taking behavior on which the model relies. Nevertheless, Brown and Matzkin do provide a restriction that can conceivably be refuted. Despite this development, a carefully stated claim

based on the SMD results on refutability remains true. That is, if the only data we have access to are at the aggregate level, general equilibrium theory does not generate refutable restrictions. This is because the Brown-Matzkin results require individual-level income or endowment vectors. Much of the intuition that would seem to follow from the SMD results is still intact. Rationality, as Arrow said, does not have aggregate implications, but now, we have to add, if only aggregate data are available.

Matters are even clearer on qualitative features of equilibrium such as local uniqueness, stability, and comparative statics. The equilibrium manifold approach employing a finite set of observations does not allow us to refute statements on these features of equilibrium. Thus many of the problematic outcomes from SMD theory remain entrenched. Not only are there no results for general configurations of the data in these areas (Nachbar 2002, 2004), we cannot test to see if an economy is poorly behaved. So we still have no progress on these aspects of the theory. In this important area, then, the intuition that general equilibrium theory is devoid of meaningfully general results remains true. It turns out that Arrow was correct to conclude that "if agents are different in unspecifiable ways, then . . . very little, if any, inferences can be made."

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